

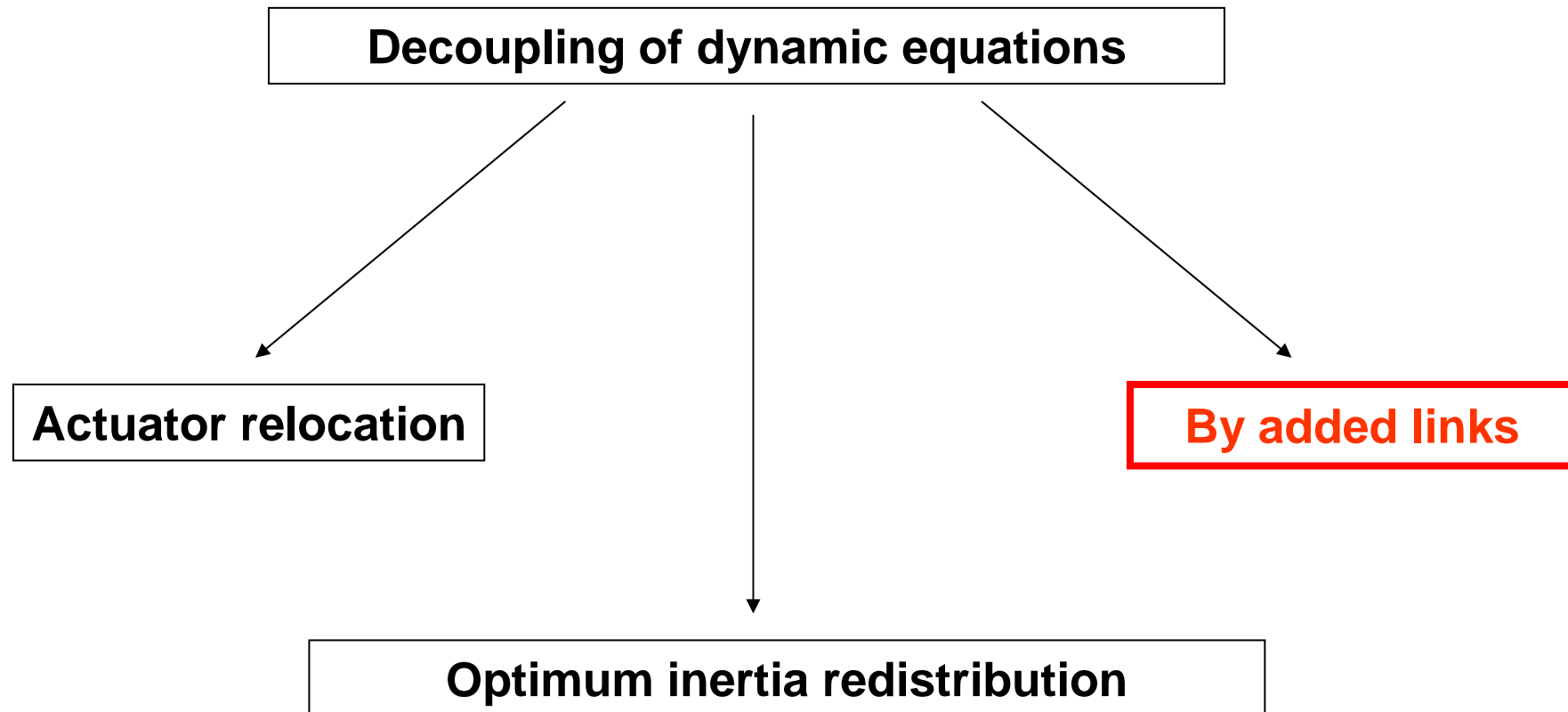
# On the Design of the Exoskeleton Arm with Decoupled Dynamics

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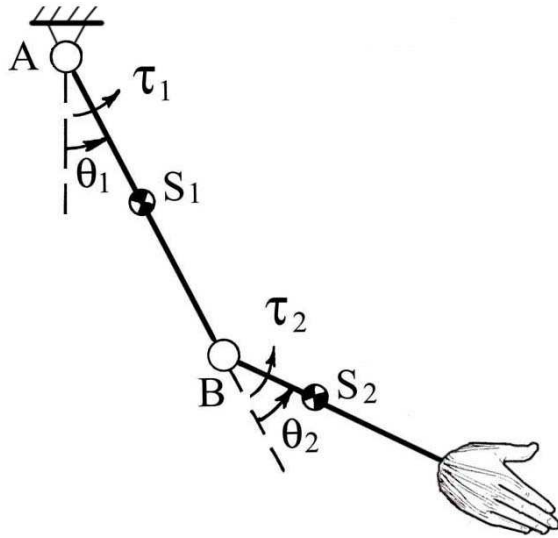


Figure 1

Coupled dynamic equations:

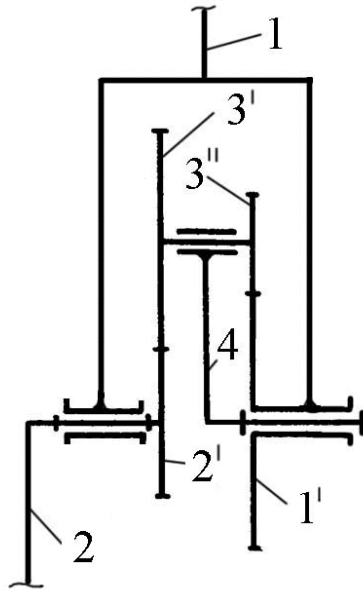
$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} D_{111} & D_{122} \\ D_{211} & D_{222} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \end{bmatrix} +$$

$$\begin{bmatrix} D_{112} & D_{121} \\ D_{212} & D_{221} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \dot{\theta}_2 \\ \dot{\theta}_1 \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

with

$$\begin{aligned} D_{11} &= m_1 l_{AS1}^2 + m_2 l_1^2 + m_2 l_{BS2}^2 + 2m_2 l_1 l_{BS2} \cos \theta_2 + I_{S1} + I_{S2}; & D_{12} &= D_{21} = \\ & m_2 l_{BS2}^2 + m_2 l_1 l_{BS2} \cos \theta_2 + I_{S2}; & D_{22} &= m_2 l_{BS2}^2 + I_{S2}; & D_{111} &= 0; & D_{122} &= \\ & -m_2 l_1 l_{BS2} \sin \theta_2; & D_{211} &= m_2 l_1 l_{BS2} \sin \theta_2; & D_{222} &= 0; & D_{112} &= D_{121} = \\ & -m_2 l_1 l_{BS2} \sin \theta_2; & D_{212} &= D_{221} = 0; & D_1 &= (m_1 l_{AS1} + m_2 l_1) g \cos \theta_1 + \\ & m_2 g l_{BS2} \cos(\theta_1 + \theta_2); & D_2 &= m_2 g l_{BS2} \cos(\theta_1 + \theta_2). \end{aligned}$$

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**Figure 2.** The epicyclic gear train added between links 1 and 2.

The Lagrangian of the manipulator with the added links:

$$L = K - P = \sum_{i=1}^4 K_i - \sum_{i=1}^4 P_i =$$

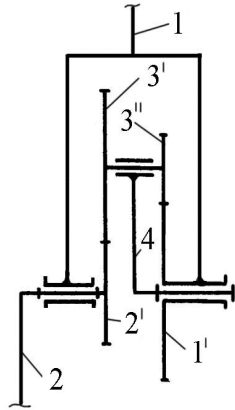
$$0.5 \left( m_1 V_{S1}^2 + m_2 V_{S2}^2 + m_3 V_{S3}^2 + m_4 V_{S4}^2 \right) +$$

$$0.5 \left[ I_{S1} \dot{\theta}_1^2 + I_{S2} (\dot{\theta}_1 + \dot{\theta}_2)^2 + I_{S3} (\dot{\theta}_2 + \dot{\theta}_3)^2 + \right.$$

$$\left. + I_{S4} (\dot{\theta}_2 + \dot{\theta}_4)^2 \right] - \sum_{i=1}^4 P_i$$

$$\sum_{i=1}^4 P_i = 0 \implies L = 0.5 \left[ \left( m_1 l_{AS1}^2 + I_{S1} + m l_1^2 \right) \dot{\theta}_1^2 + I_{S2} (\dot{\theta}_1 + \dot{\theta}_2)^2 + I_{S3} (\dot{\theta}_1 + \dot{\theta}_3)^2 + I_{S4} (\dot{\theta}_1 + \dot{\theta}_4)^2 \right]$$

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The transmission ratio in the epicyclic gear train:

$$\frac{\dot{\theta}_2}{\dot{\theta}_4} = 1 - \frac{Z_{3'}Z_{1'}}{Z_{2'}Z_{3''}}$$

**Figure 2.** The epicyclic gear train added between links 1 and 2.

$$Z_{3'} = 2Z_{2'}$$

$$Z_{1'} = Z_{3''}$$



$$\dot{\theta}_2 = -\dot{\theta}_4$$

$$\dot{\theta}_3 = 2\dot{\theta}_4 = -2\dot{\theta}_2$$

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The Lagrangian of the manipulator with the added links can be rewritten as:

$$L = 0.5 \left( k_1 \dot{\theta}_1^2 + k_2 \dot{\theta}_2^2 + k_3 \dot{\theta}_1 \dot{\theta}_2 \right)$$

where

$$k_1 = m_1 l_{AS1}^2 + m l_1^2 + I_{S1} + I_{S2} + I_{S3} + I_{S4}$$

$$k_2 = I_{S2} + 4I_{S3} + I_{S4}$$

$$k_3 = 2(I_{S2} - 2I_{S3} - I_{S4})$$

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if  $k_3 = 0$ , i.e.  $I_{S2} = 2I_{S3} + I_{S4}$ ,



$$L = 0.5(k_1\dot{\theta}_1^2 + k_2\dot{\theta}_2^2)$$



$$\left. \begin{aligned} \tau_1 &= k_1\ddot{\theta}_1 \\ \tau_2 &= k_2\ddot{\theta}_2 \end{aligned} \right\} \text{Dynamic decoupling}$$

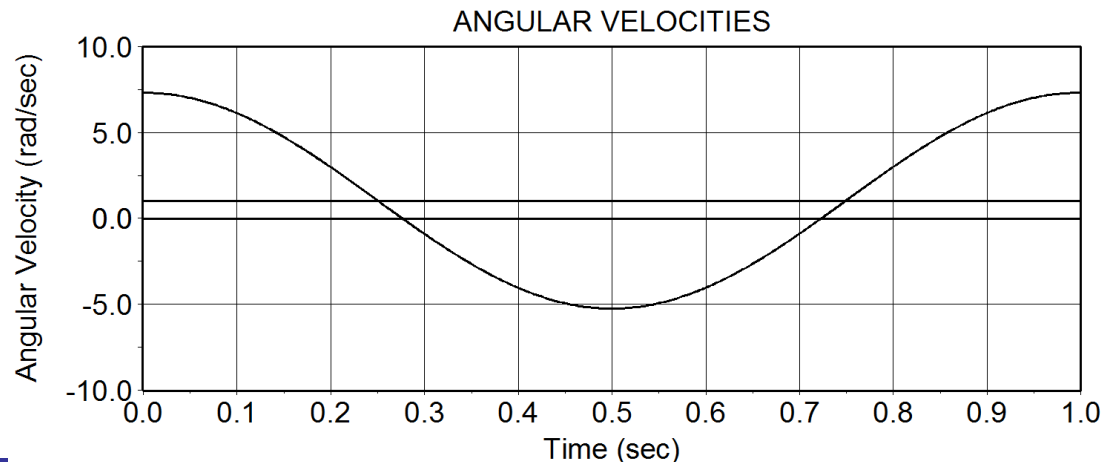
## On the Design of the Exoskeleton Arm with Decoupled Dynamics

- Numerical simulations

The exoskeleton arm parameters with decoupled dynamic equations are the following:  $m_1 = 2kg$  ,  $m_2 = 8kg$  ,  $I_{S1} = I_{S2} = 0.1kgm^2$  ,  $l_{AS1} = 0.1m$  ,  $l_{BS2} = 0$  (taking into account that the epicyclic gear train is mainly a balanced system),  $I_{S3} = 0.03kgm^2$  ,  $I_{S4} = 0.04kgm^2$  (i.e.  $I_{S2} = 2I_{S3} + I_{S4}$ ).

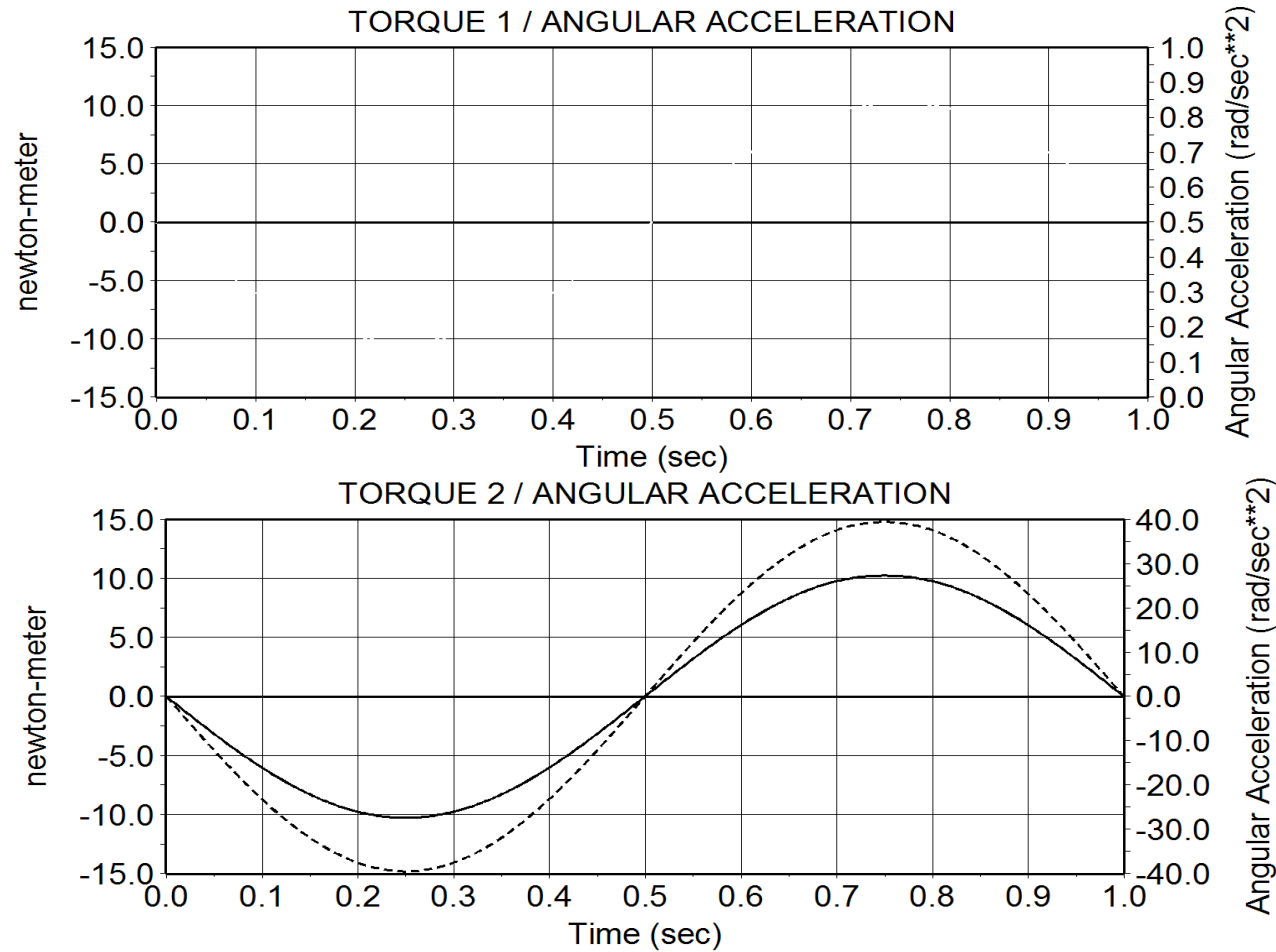
To better show the dynamic decoupling of motion equations the following input laws have been chosen:  $\theta_1 = 60t$  and  $\theta_2 = \sin(2\pi t)$  with  $0 \leq t \leq 1s$  .

Thus, the angular accelerations are  $\ddot{\theta}_1 = 0$  and  $\ddot{\theta}_2 = -4\pi^2 \sin(2\pi t)$  .





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**THANK YOU FOR YOUR ATTENTION**

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