

# Strategy to lock the knee of exoskeleton stance leg: study in the framework of ballistic walking model

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- To design a **wearable assist device** for human.
  - To improve daily life for patients or elderly
  - To avoid the musculoskeletal disorders for industrial workers

Desired performances:

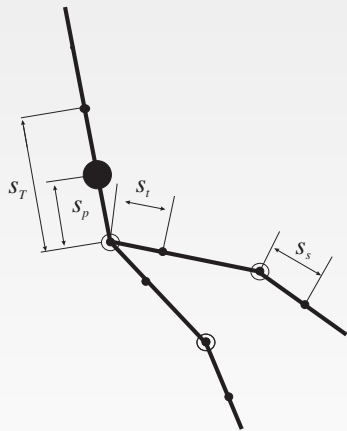
- To be able **to compensate a part of the loads** due the human's weight.
- To have an assist device **with an energetic autonomy**.
- **Good adaptation to the shape** of human's body.

# Statement of the problem

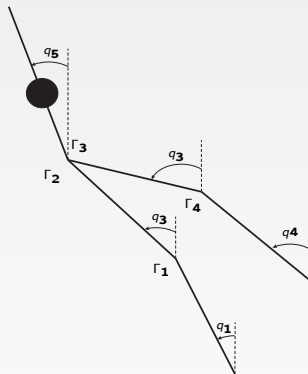
- To consider a five-link planar Biped **with a strapped exoskeleton**.
- There are **not any actuators** in our exoskeleton.
- To define a ballistic walking gait of the biped alone **without any assist** device for the transport of a load.
- During ballistic walking of the biped with exoskeleton the knee of the stance leg of the exoskeleton (and as a consequence of the biped) **is locked**.
- Walking of the biped consists of alternating single- and **instantaneous double-support phases**.
- At the instant of this phase, the knee of the previous swing leg is locked and the knee of the **previous stance leg is unlocked**.
- To compare the energy consumption of the biped **alone and with its exoskeleton**.



# The geometrical structure of the biped and its wearable assist device.



(a)



(b)

- **seven** generalized coordinates  $\mathbf{x} = [q_1, q_2, q_3, q_4, q_5, x, y]^T$ .

# Physical parameters.

	Mass (kg)	Length (m)	Inertia moment (kg·m <sup>2</sup> )	center of mass (m)
Human shin	$m_s = 4.6$	$l_s = 0.55$	$I^s = 0.0521$	$s_s = 0.324$
Human thigh	$m_t = 8.6$	$l_t = 0.45$	$I^t = 0.75$	$s_t = 0.18$
Human trunk	$m_T = 48.6$	$l_T = 0.75$	$I^T = 11.3$	$s_T = 0.386$
Exoskeleton shin	$m_1 = 1.0$	$l_1 = 0.497$	$I^1 = 0.0260$	$s_s = 0.324$
Exoskeleton thigh	$m_2 = 2.0$	$l_2 = 0.41$	$I^2 = 0.0354$	$s_t = 0.18$
Exoskeleton trunk	$m_3 = 8.0$	$l_3 = 0.75$	$I^3 = 0.3817$	$s_T = 0.386$

## Equations of the motion of the biped in single support

$$\mathbf{A}(\mathbf{x})\ddot{\mathbf{x}} + \mathbf{h}(\mathbf{x}, \dot{\mathbf{x}}) = \mathbf{D}\Gamma + \mathbf{J}_{\mathbf{r}_1}^\top \mathbf{r}_1 + \mathbf{J}_{\mathbf{r}_2}^\top \mathbf{r}_2, \quad (1)$$

with the constraint equations,

$$\mathbf{J}_{\mathbf{r}_i} \ddot{\mathbf{x}} + \dot{\mathbf{J}}_{\mathbf{r}_i} \dot{\mathbf{x}} = \mathbf{0} \quad \text{for } i = 1 \text{ or/and } 2. \quad (2)$$

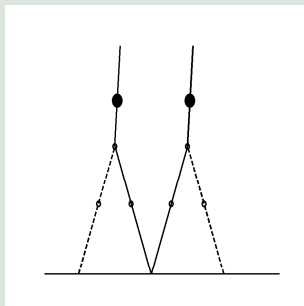
The joint variables  $\theta_i$  for  $i = (1, 2, 3, 4)$  as functions of the generalized coordinates are:

$$\theta_1 = q_2 - q_1, \quad \theta_2 = q_5 - q_2, \quad \theta_3 = q_5 - q_3, \quad \theta_4 = q_3 - q_4. \quad (3)$$

# Definition of the ballistic motion

## Statement of the problem

- Let  $\mathbf{x}(0)$  be an initial configuration  $t = 0$ .
- Let  $\mathbf{x}(T)$  be an final configuration  $t = T$ .



# Definition of the ballistic motion

ballistic motion be in single support:  $\Gamma = 0$ .

$$\mathbf{A}(\mathbf{x})\ddot{\mathbf{x}} + \mathbf{h}(\mathbf{x}, \dot{\mathbf{x}}) = [\mathbf{D}_1 \quad \mathbf{D}_2 \quad \mathbf{D}_3 \quad \mathbf{D}_4] \begin{bmatrix} \Gamma_1 \\ \mathbf{0}_{3 \times 1} \end{bmatrix} + \mathbf{J}_{\mathbf{r}_1}^T \mathbf{r}_1,$$

with the constraint equation for the stance leg tip fixed on the ground:

$$\mathbf{J}_{\mathbf{r}_1} \ddot{\mathbf{x}} + \dot{\mathbf{J}}_{\mathbf{r}_1} \dot{\mathbf{x}} = \mathbf{0},$$

and with the constraint equation for the knee of the stance leg locked in the swing phase:

$$\mathbf{D}_1^T \ddot{\mathbf{x}} = 0.$$

- problem: Which velocity vector  $\dot{\mathbf{x}}(0)$  such that  $\mathbf{x}(t)$  starting from  $\mathbf{x}(0)$  reaches  $\mathbf{x}(T)$

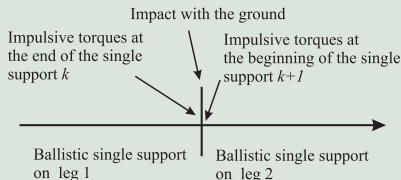
⇒ A boundary value problem solved using a Newton method





## decomposition of the impulsive impact.

- After impact the velocity of the biped has to be equal to the founded initial velocity.  $\Rightarrow$  **impulsive** impact.



- Let  $\dot{\mathbf{x}}^b$  be the final velocity vector of the current ballistic swing,
- Let  $\dot{\mathbf{x}}^a$  be the initial velocity vector of the next ballistic swing.

# The structure of the **instantaneous** double support

First sub-phase: Impulsive phase.

$$\mathbf{A}[\mathbf{x}(T)](\dot{\mathbf{x}}^- - \dot{\mathbf{x}}^b) = [\mathbf{D}_1 \quad \mathbf{D}_2 \quad \mathbf{D}_3 \quad \mathbf{D}_4] \begin{bmatrix} \mathbf{l}_1^- \\ \mathbf{l}_{3 \times 1}^- \end{bmatrix} + \mathbf{J}_{\mathbf{r}_1}^\top \mathbf{l}_{\mathbf{r}_1}^-. \quad (4)$$

$$\mathbf{J}_{\mathbf{r}_1} \dot{\mathbf{x}}^- = \mathbf{0}_{2 \times 1} \quad (5)$$

The velocity of the inter-link angle of the stance (hind) leg after first sub-phase remains zero, therefore

$$\mathbf{D}_1^\top \dot{\mathbf{x}}^- = 0. \quad (6)$$

$\mathbf{l}_{3 \times 1}^-$  applied by human.

# The structure of the **instantaneous** double support

## Second sub-phase: Passive impact.

Knee of the leg 1 is **unlocked**, knee of the leg 2 is **locked** with  $I_4$ . The **second** sub-phase: passive impact. The stance leg lifts off the ground (Hypothesis).

$$\mathbf{A}(\dot{\mathbf{x}}^+ - \dot{\mathbf{x}}^-) = [\mathbf{D}_1 \quad \mathbf{D}_2 \quad \mathbf{D}_3 \quad \mathbf{D}_4] \begin{bmatrix} \mathbf{0}_{3 \times 1} \\ I_4 \end{bmatrix} + \mathbf{J}_{r_2}^T \mathbf{I}_{r_2} \quad (7)$$

The associate equation:

$$\mathbf{J}_{r_2} \dot{\mathbf{x}}^+ = \mathbf{0}_{2 \times 1} \quad (8)$$

To take into account the locking of the knee of the leg 2, we have to complete equations (7) and (8) with equation:

$$\mathbf{D}_4^T \dot{\mathbf{x}}^+ = 0. \quad (9)$$

## Third sub-phase: Impulsive impact.

$$\mathbf{A}(\dot{\mathbf{x}}^a - \dot{\mathbf{x}}^+) = [\mathbf{D}_1 \quad \mathbf{D}_2 \quad \mathbf{D}_3 \quad \mathbf{D}_4] \begin{bmatrix} \mathbf{I}_{3 \times 1}^+ \\ \mathbf{I}_4^+ \end{bmatrix} + \mathbf{J}_{\mathbf{r}_2}^\top \mathbf{I}_{\mathbf{r}_2}^+ \quad (10)$$

There are **27** scalar equations to find **29** unknown variables, which are the components of the vectors and scalars:

- $\dot{\mathbf{x}}^-(7 \times 1)$ ,  $\mathbf{l}_1^-$ ,  $\mathbf{I}_{3 \times 1}^-(\mathbf{l}_2^-, \mathbf{l}_3^-, \mathbf{l}_4^-)^\top$ ,  $\mathbf{I}_{\mathbf{r}_1}^-(2 \times 1)$  (for the *first* sub-phase),
- $\dot{\mathbf{x}}^+(7 \times 1)$ ,  $\mathbf{l}_4$ ,  $\mathbf{I}_{\mathbf{r}_2}(2 \times 1)$  (for the *second* sub-phase),
- $\mathbf{I}_{3 \times 1}^+(\mathbf{l}_1^+, \mathbf{l}_2^+, \mathbf{l}_3^+)^\top$ ,  $\mathbf{l}_4^+$  and  $\mathbf{I}_{\mathbf{r}_2}^+(2 \times 1)$  (for the *third* sub-phase).

With the impulsive torques, the cost functional is

$$W = \sum_{i=2}^4 \int_{T^-}^T |\Gamma_i^-(t) \dot{\theta}_i(t)| dt + \sum_{i=1}^3 \int_T^{T^+} |\Gamma_i^+(t) \dot{\theta}_i(t)| dt$$
$$W = \sum_{i=2}^4 W_i^- + \sum_{i=1}^3 W_i^+ \quad (11)$$

The energy due to the impulsive torques becomes [Formal 82]

$$W = \sum_{i=2}^4 W_i^- + \sum_{i=1}^3 W_i^+ \text{ with:}$$

Values  $W_i^-$  ( $i = 2, 3, 4$ ) are calculated as follows:

$$W_i^- = \left| l_i^- \frac{\dot{\theta}_i^b + \dot{\theta}_i^-}{2} \right| \quad \text{if } \dot{\theta}_i^b \dot{\theta}_i^- \geq 0 \quad i = 2, 3, \text{ and } 4,$$

$$W_i^- = \left| l_i^- \frac{(\dot{\theta}_i^b)^2 + (\dot{\theta}_i^-)^2}{2 [\dot{\theta}_i^- - \dot{\theta}_i^b]} \right| \quad \text{if } \dot{\theta}_i^b \dot{\theta}_i^- < 0 \quad i = 2, 3, \text{ and } 4,$$

$$\dot{\theta}_2^b = \dot{q}_5^b - \dot{q}_2^b, \quad \dot{\theta}_3^b = \dot{q}_5^b - \dot{q}_3^b, \quad \dot{\theta}_4^b = \dot{q}_3^b - \dot{q}_4^b,$$

$$\dot{\theta}_2^- = \dot{q}_5^- - \dot{q}_2^-, \quad \dot{\theta}_3^- = \dot{q}_5^- - \dot{q}_3^-, \quad \dot{\theta}_4^- = \dot{q}_3^- - \dot{q}_4^-$$

(12)



The energy due to the impulsive torques becomes [Formal 82]

$$W = \sum_{i=2}^4 W_i^- + \sum_{i=1}^3 W_i^+ \text{ with:}$$

Values  $W_i^+$  ( $i = 1, 2, 3$ ) are calculated using analogous formulas:

$$W_i^+ = \left| I_i^+ \frac{\dot{\theta}_i^+ + \dot{\theta}_i^a}{2} \right| \quad \text{if } \dot{\theta}_i^+ \dot{\theta}_i^a \geq 0 \quad i = 1, 2, \text{ and } 3,$$

$$W_i^+ = \left| I_i^+ \frac{(\dot{\theta}_i^+)^2 + (\dot{\theta}_i^a)^2}{2 [\dot{\theta}_i^a - \dot{\theta}_i^+]} \right| \quad \text{if } \dot{\theta}_i^+ \dot{\theta}_i^a < 0 \quad i = 1, 2, \text{ and } 3.$$

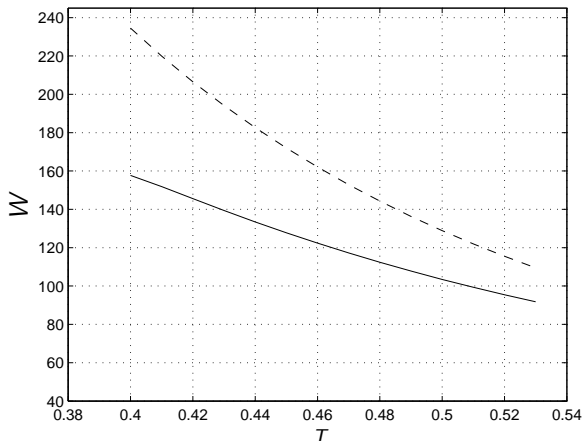
$$\dot{\theta}_1^+ = \dot{q}_2^+ - \dot{q}_1^+, \quad \dot{\theta}_2^+ = \dot{q}_5^+ - \dot{q}_2^+, \quad \dot{\theta}_3^+ = \dot{q}_5^+ - \dot{q}_3^+,$$

$$\dot{\theta}_1^a = \dot{q}_2^a - \dot{q}_1^a, \quad \dot{\theta}_2^a = \dot{q}_5^a - \dot{q}_2^a, \quad \dot{\theta}_3^a = \dot{q}_5^a - \dot{q}_3^a$$

(13)



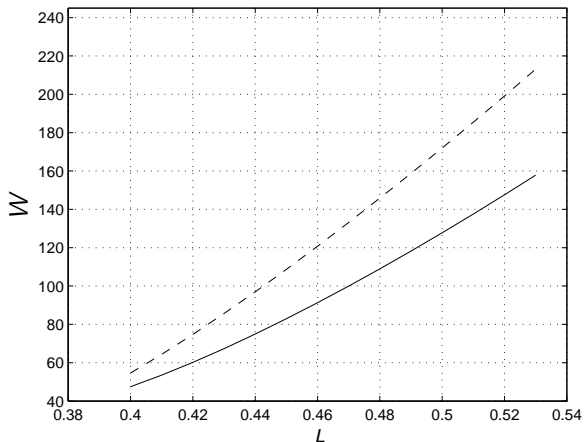
# Energy cost as a function of $T$ . (1/2)



For human with the exoskeleton (diamond), and for human alone (circle).



## Energy cost as a function of $L$ . (2/2)



For human with the exoskeleton (diamond), and for human alone (circle).

## Conclusion

- The planar five-link anthropomorphic bipedal mechanism is considered as a mechanical model of a human walking.
- The links of the exoskeleton are strongly strapped to the corresponding links of the biped.
- A passive exoskeleton without any sources of energy and actuators; the knee of the stance leg of our exoskeleton (and as consequence of the human) is locked using mechanical brake device, but the knee of the swing leg is unlocked.
- It is shown theoretically the efficiency of mentioned above passive exoskeleton for human carrying a load.

## Perspectives

- To add feet to our biped.
- to extend to 3D motions.
- To extend this work with optimal walking gaits